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For the use of the junior class of mathematics in the
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ELEMENTS

OF

PLANE TRIGONOMETRY

FOR THE USE OF THE JUNIOR CLASS OF MATHEMATICS IN THE UNIVERSITY OF GLASGOW.

BY

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PREFACE.

Some apology is required for adding another to the long list of books on Trigonometry. My excuse is that during twenty years' experience I have not found any published book exactly suiting the wants of my Students. In conducting a Junior Class by regular progressive steps from Euclid and Elementary Algebra to Trigonometry, I have had to fill up by oral instruction the gap between the Sixth Book of Euclid and the circular measurement of Angles; which is not satisfactorily bridged by the propositions of Euclid's Tenth and Twelfth Books usually *supposed to be* learned; nor yet by demonstrations in the modern books on Trigonometry, which mostly follow Woodhouse; while the Appendices to Professor Robert Simson's Euclid in the editions of Professors Playfair and Wallace of Edinburgh, and of Professor James Thomson of Glasgow, seemed to me defective for modern requirements, as not sufficiently connected with Analytical Trigonometry.

What I felt the want of was a short Treatise, to be used as a Text Book after the Sixth Book of Euclid had been learned and some knowledge of Algebra acquired, which should contain satisfactory demonstrations of the propositions to be used in teaching Junior Students the Solution of Triangles, and should at the same time lay a solid foundation for the study of Analytical Trigonometry.

This want I have attempted to supply by applying, in the first Chapter, Newton's Method of Limits to the mensuration of circular arcs and areas; choosing that method both because it is the strictest and the easiest, and because I think the Mathematical Student should be early introduced to the method.

The succeeding Chapters are devoted to an exposition of the nature of the Trigonometrical ratios, and to the demonstration by geometrical constructions of the principal propositions required for the Solution of Triangles. To these I have added a general explanation of the applications of these propositions in Trigonometrical Surveying: and I have concluded with a proof of the formulæ for the sine and cosine of the sum of two angles treated (as it seems to me they should be) as examples of the Elementary Theory of Projection. Having learned thus much the Student has gained a knowledge of Trigonometry as originally understood, and may apply his knowledge in Surveying; and he has also reached a point from which he may advance into Analytical Trigonometry and its use in Natural Philosophy.

Thinking that others may have felt the same want as myself, I have published the Tract instead of merely printing it for the use of my Class.

H. B.

ELEMENTS

OF

PLANE TRIGONOMETRY.

TRIGONOMETRY (from τρίγωνον, triangle, and μετρέω, I measure) is the science of the numerical relations between the sides and angles of triangles.

This Treatise is intended to demonstrate, to those who have learned the principal propositions in the first six books of Euclid, so much of Trigonometry as was originally implied in the term, that is, how from given values of some of the sides and angles of a triangle to calculate, in the most convenient way, all the others.

A few propositions supplementary to Euclid are premised as introductory to the propositions of Trigonometry as usually understood.

CHAPTER I.

OF THE MENSURATION OF THE CIRCLE.

DEF. 1. A magnitude or ratio, which is fixed in value by the conditions of the question, is called a CONSTANT.

DEF. 2. A magnitude or ratio, which is not fixed in value by the conditions of the question and which is conceived to change its value by lapse of time, or otherwise, is called a VARIABLE.

DEF. 3. If a variable shall be always less than a given constant, but shall in time become greater than any less constant, the given constant is the SUPERIOR LIMIT of the variable: and if the variable shall be always greater than a given constant but in time shall become less than any greater constant, the given constant is the Inferior Limit of the variable.

Lemma. *If two variables are at every instant equal their limits are equal.*

For if the limits be not equal, the one variable shall necessarily in time become greater than the one limit and less than the other, while at the same instant the other variable shall be greater than both limits or less than both limits, which is impossible, since the variables are always equal.

Def. 4. Curvilinear segments are *similar* when, if on the chord of the one as base any triangle be described with its vertex in the arc, a similar triangle with its vertex in the other arc can always be described on its chord as base; and the arcs are Similar Curves.

Cor. 1. Arcs of circles subtending equal angles at the centres are similar curves.

Cor. 2. If a polygon of any number of sides be inscribed in one of two similar curves, a similar polygon can be inscribed in the other.

Def. 5. Let a number of points be taken in a terminated curve line, and let straight lines be drawn from each point to the next, then if the number of points be conceived to increase and the distance between each two to diminish continually, the extremities remaining fixed, the limit of the sum of the straight lines is called the Length of the Curve.

Prop. I. *The lengths of similar arcs are proportional to their chords.*

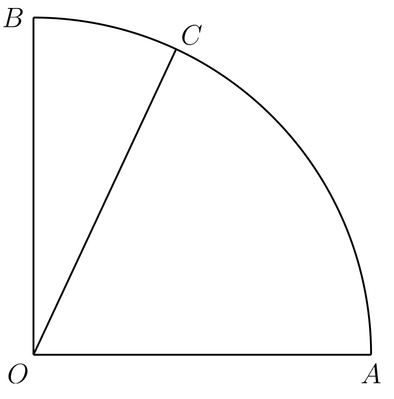
For let any number of points be taken in the one and the points be joined by straight lines so as to inscribe a polygon in it, and let a similar polygon be inscribed in the other, the perimeters of the two polygons are proportional to the chords, or the ratio of the perimeter of the one to its chord is equal to the ratio of the perimeter of the other to its chord. Then if the number of sides of the polygons increase these two ratios vary but remain always equal to each other, therefore (Lemma) their limits are equal. But the limit of the ratio of the perimeter of the polygon to the chord is (Def. 5) the ratio of the length of the curve to its chord, therefore the ratio of the length of the one curve to its chord is equal to the ratio of the length of the other curve to its chord, or the lengths of similar finite curve lines are proportional to their chords.

COR. 1. Since semicircles are similar curves and the diameters are their chords, the ratio of the semi-circumference to the diameter is the same for all circles.

If this ratio be denoted, as is customary, by $\frac{\pi}{2}$, then numerically

$$\text{the circumference} \div \text{the diameter} = \pi,$$
$$\text{and the circumference} = 2\pi R.$$

COR. 2. The angle subtended at the centre of a circle by an arc equal to the radius is the same for all circles. For if AC be the arc equal to the radius, and AB the arc subtending a right angle, then by Euclid VI. 33



$$AOC : AOB :: AC : AB.$$

But AB is a fourth of the circumference $= \frac{\pi R}{2}$;

$$\text{therefore } AOC : \text{a right angle} :: R : \frac{\pi R}{2} :: 2 : \pi$$

$$\text{or numerically } AOC = \frac{2}{\pi} \times \text{a right angle},$$

that is the angle subtended by an arc equal to the radius is a fixed fraction of a right angle.

Prop. II. *The areas of similar segments are proportional to the squares on their chords.*

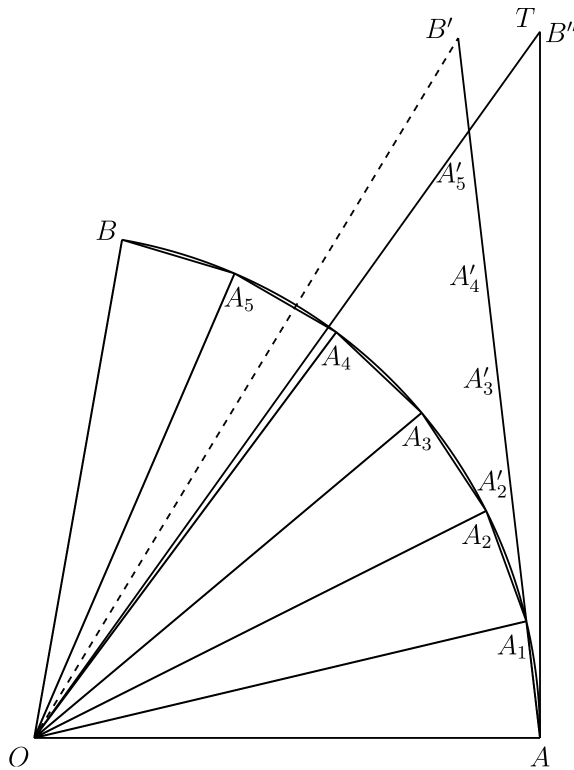
For, if similar polygons of any number of sides be inscribed in the similar segments, they are to one another in the duplicate ratio of the chords, or, alternately, the ratio of the polygon inscribed in the one segment to the square on its chord is the same as the ratio of the similar polygon in the other segment to the square on its chord. Now conceive the polygons to vary by the number of sides increasing continually while the two polygons remain always similar, then the variable ratios of the polygons to the squares on the chords always remain equal, and therefore their limits are equal (Lemma); and these limits are obviously the ratios of the areas of the segments to the squares on the chords, which ratios are therefore equal.

Cor. Circles are to one another as the squares of their diameters.

Note. From Prop. II. and III. it is obvious that "The corresponding sides, whether straight or curved, of similar figures, are proportionals; and their areas are in the duplicate ratio of the sides." (Newton, *Princip.* I. Sect. i. Lemma v.)

Prop. III. *The area of any circular sector is half the rectangle contained by its arc and the radius of the circle.*

Let AOB be a sector. In the arc AB take any number of equidistant points $A_1, A_2, \ldots\ldots A_n$, and join $AA_1, A_1A_2, \ldots\ldots A_nB$. Produce AA_1, and along it take parts $A_1A_2', A_2'A_3', \ldots\ldots A_n'B'$ equal to $A_1A_2, A_2A_3, \ldots\ldots A_nB$ respectively: so that AB' is equal to the polygonal perimeter $AA_1A_2 \ldots\ldots A_nB$; then if the number of points A_1, A_2, &c., be conceived to increase continually, the limit of AB' is the arc AB.



Now through A draw the line AT at right angles to OA, then as the number of points increases continually, the angle TAB' shall diminish continually, and shall in time become less than any finite angle, and the limit of the position of AB' shall be AB'' measured along AT, where AB'' is equal in length to the arc AB.

Join OA'_1, OA'_2, $OA'_3, \ldots\ldots OA'_n$ and the triangles OA_1A_2, OA_2A_3, $OA_3A_4, \ldots\ldots OA_nB$ are equal, each to each, to $OA_1A'_2$, $OA'_2A'_3$, $OA'_3A'_4, \ldots\ldots OA'_nB$, for the perpendiculars from O on the sides of the polygon are all equal to the perpendicular on AB'; therefore the variable triangle OAB' is always equal to the variable polygon

$OAA_1A_2 \ldots\ldots A_nB$; therefore their limits are equal. But the limit of the triangle OAB' is OAB'' and the limit of the polygon is the sector OAB; therefore the sector AOB is equal to the triangle OAB'', which is half the rectangle OA, AB'', or half the rectangle contained by the radius and the arc.

Hence the area of a circle $= \dfrac{1}{2} R \times$ circumference $= \pi R^2$ and the ratio of the circle to the square on its diameter is $= \dfrac{\pi}{4}$.

Prop. IV. *Any line, whether curved or polygonal, which is convex throughout (that is, which can be cut by a straight line in only two points), is less than any line, curved or polygonal, which envelopes it from one extremity to the other**.

For the enveloping line is obviously greater than the sum of any number of straight lines drawn as in Def. 5, and therefore is greater than the limit of that sum, that is, than the length of the curve.

Cor. Hence two straight lines, touching at its extremities any circular arc less than a semicircle, are together greater than the arc.

Prop. V. *If circles be inscribed in and described about two regular polygons of the same perimeter, the second of which has twice as many sides as the first, then* (1) *the radius of the circle inscribed in the second is an arithmetic mean between* (*i.e. is half the sum of*) *the radius of the circle inscribed in and the radius of the circle described about the first; and* (2) *the radius of the circle described about the second is a mean proportional between the radius of the circle inscribed in the second, and the radius of the circle described about the first.*

Let BB' be a side of the first polygon, C the centre of the circle described about it.

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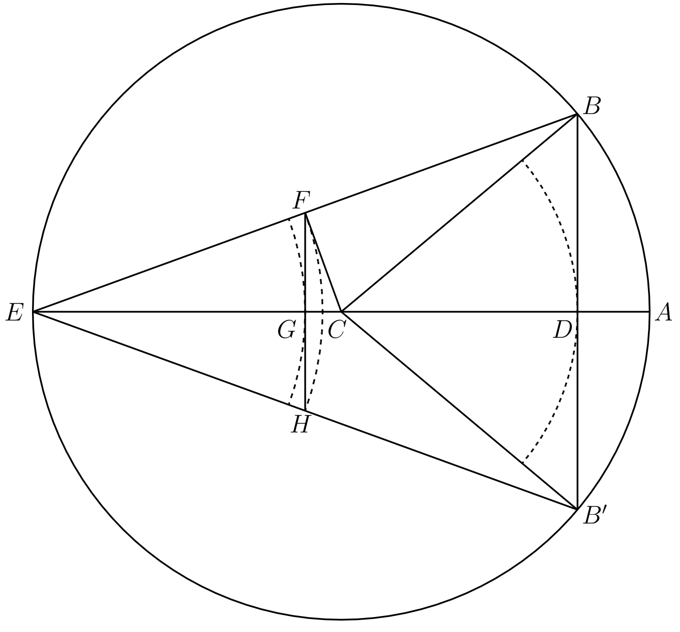
*This enunciation is taken from Legendre, *Elements de Geometrie*, 12me ed. Liv. iv. Prop. ix., but the demonstration is different.

From C as centre with CB as radius describe the circle $BB'E$.

Draw ECA a diameter perpendicular to BB' and therefore bisecting it in D.



Join EB, EB'. Draw CF perpendicular to EB, and FGH perpendicular to EA.

Then, because the angle BEB' is half of BCB', and FH is half of BB', for FH bisects EB and EB'; therefore $FH =$ the side of the second polygon, and $FEH =$ the angle it subtends at the centre.

Therefore EF is the radius of the circle described about the second polygon, and EG the radius of the circle inscribed in it.

And CD, CB are the radii of the circles inscribed in and described about the first polygon.

But EG is half of ED, that is, half of EC (or CB) and CD together, that is the radius of the circle inscribed in the second polygon is the arithmetic mean of the radii of the circles inscribed in and described about the first polygon.

Again, because the triangles EFG, ECF are similar,

$$EC : EF :: EF : EG,$$

that is, the radius of the circle described about the second polygon is a mean proportional to that of the circle described about the first and that of the circle inscribed in the second.

Cor. Hence the ratio of the circumference of a circle to its diameter (or π) can be calculated to any degree of accuracy.

For let R, R' be the radii of the circles described about, and r, r' of those inscribed in, the first and second polygon respectively, then

$$r' = \frac{R+r}{2}; \quad R' = \sqrt{r' \cdot R}.$$

From these it will be easy to calculate successively the radii of circles inscribed in and described about isoperimetrical polygons of 2, 4, 8, 16, 32, &c. times the number of sides of a given regular polygon.

Then, if the radii and perimeter of a regular polygon of any number of sides be known, by making it the first polygon of the series and calculating the radii for a sufficient number of succeeding polygons, we can calculate the value of π (the ratio of the circumference of a circle to its diameter) to any degree of accuracy. For since the perimeter of each polygon will lie between the circumference of its inscribed and circumscribed circles if R and r be the radii for any polygon of the series, we shall have $2\pi R$ greater, and $2\pi r$ less than p, the common perimeter of all the polygons. Therefore π is intermediate to $\dfrac{p}{2R}$ and

$\dfrac{p}{2r}$, and, by doubling the number of sides of the polygon sufficiently, R and r can be made to differ as little as we please, and therefore π can be calculated as accurately as desired.

The calculation is not very laborious. Thus, if we begin from a square, each side of which is the unit, we have $r_1 = 0.5$ and

$$R_1 = \sqrt{.5} = 0.7071067812.$$

Then

$$r_2 = \frac{0.5 + 0.7071067812}{2} = 0.6035533906,$$

and

$$\begin{aligned} R_2 &= \sqrt{.7071067812 \times .6035533906} \\ &= 0.6532814824. \end{aligned}$$

In like manner the radii of circles inscribed in and described about polygons of 16, 32, 64, 128, &c. sides with the same perimeter (viz. 4) are successively found by alternately taking arithmetic and geometric means.

Stopping at the polygon of 1024 sides, it appears that

$$\frac{2000000}{636621} < \pi < \frac{2000000}{636617},$$

i.e. $3.14158 < \pi < 3.14160$.

It may however be shewn (see Appendix) that, when the difference between R and r is small, $\frac{1}{3}(r + 2R)$ is a very near approximation to the limit of both radii, and that therefore π may be taken $= \dfrac{\frac{1}{2}p}{\frac{1}{3}(r + 2R)}$ with great accuracy.

No. of sides of the Polygon.	Radius of Inscribed Circle $= r$.	Radius of Circumscribing Circle $= R$.
4	.5000000000	.7071067812
8	.6035533906	.6532814824
16	.6284174365	.6407288619
32	.6345731492	.6376435773
64	.6361083633	.6368755077
128	.6364919355	.6366836927
256	.6365878141	.6366357516
512	.6366117828	.6366237671
1024	.6366177750	.6366207710
&c.	&c.	&c.

Taking the radii for 1024 sides

$$\frac{r+2R}{3} = \frac{1}{3}\left\{\begin{array}{r} .6366177750 \\ 1.2732415420 \end{array}\right\} = \frac{1.9098593170}{3} = .6366197723,$$

which will be found to be ten decimals of the radius of the circle inscribed in a polygon of 262144 and every greater number of sides if the table be continued.

Thus we may take

$$\begin{aligned} \pi &= \frac{20000000000}{6366197723}, \\ \text{or} \quad &= 3.141592654. \end{aligned}$$

By the method of "continued fractions" it will be found that $\frac{22}{7}$ and $\frac{355}{113}$ are nearer approximations to the value of π than any simpler fractions.

Of these $\dfrac{22}{7}$ $(= 3.14)$ is the approximation discovered by Archimedes (killed, it is said, at the siege of Syracuse, B.C. 212); and the approximation $\dfrac{355}{113}$ $(= 3.14159)$ was given by Adrian Metius of Alkmaer (died A.D. 1636)*.

*This simple and elegant elementary method of approximating to π is taken from Leslie's *Geometry*, V. 20; compare Legendre, *Geometrie*, IV. 14 and 16.

CHAPTER II.

OF THE AREA OF A TRIANGLE AND OF THE INSCRIBED CIRCLE.

PROP. I. *A triangle is equal to the rectangle contained by its semi-perimeter and the radius of the inscribed circle.*

Let ABC be the triangle. Bisect the angles by the lines AO, BO, CO, meeting (Euclid IV. 4) in O, the centre of the inscribed circle.

Then the triangle ABC is made up of the triangles BOC, COA, AOB, each of which stands on one of the sides, as base, with its altitude equal to the radius of the inscribed circle. Therefore the whole triangle ABC is equal to a triangle having the sum of the three sides (or the perimeter) for base and the radius of the inscribed circle for altitude; or to the rectangle having the semi-perimeter for base and the radius of the inscribed circle for altitude.

Scholium. The two tangents from each angle to the inscribed circle are equal: hence, if three tangents, one from each angle, be taken, their sum is the semi-perimeter, and therefore a tangent from one of the angles, together with the side opposite that angle, is equal to the semi-perimeter.

Let the sides opposite the angles A, B, C be represented numerically by a, b, c; the semi-perimeter by s, and the radius of the inscribed circle by r.

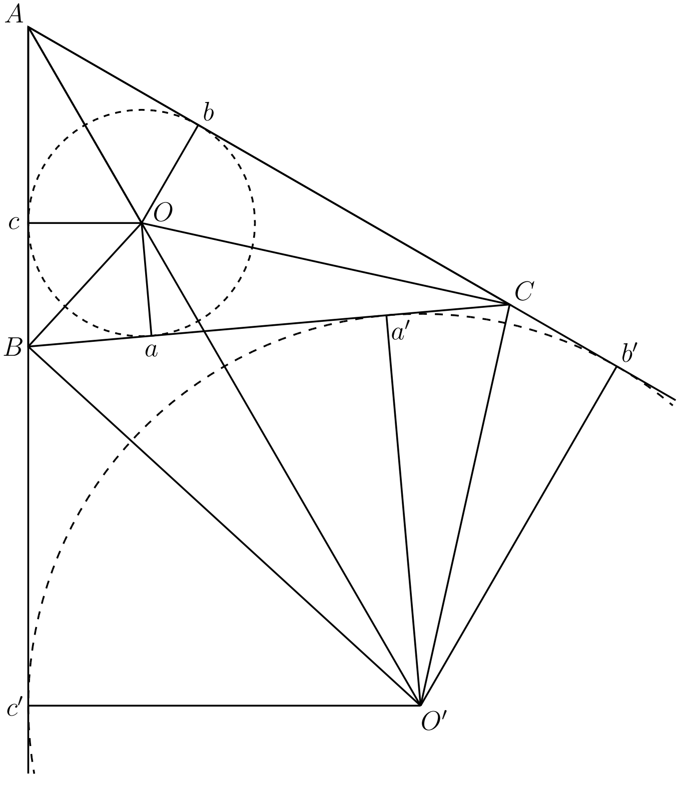
$$\text{Then, numerically, the Area} = rs.$$

$$\text{And } Ab = Ac = s - a, \quad Bc = Ba = s - b, \quad Ca = Cb = s - c.$$

DEF. Let two of the sides of the triangle ABC be produced, and a circle described touching the two produced sides and the third side.

The circle is said to be *excribed** on the third side.

Prop. II. *A triangle is equal to the rectangle contained by the radius of the circle excribed on one of its sides and the tangent from the opposite angle to the inscribed circle.*



Let ABC be the triangle. Bisect the angle A and the exterior angles at B and C by the lines AO', BO', CO', which will meet in the centre

*This word is often spelled "*escribed*" improperly. The Latin word is exscribo, but the English usage is to elide the *s* in such cases, as *expect* from exspecto, *expatiate* from exspatior, *extinguish* from exstinguo. No one ever proposed to emend these words into *espect*, *espatiate*, and *estinguish*. Why then *escribe?*

of the excribed circle*. Draw perpendiculars $O'a'$, Ob', $O'c'$ on the three sides. Then these perpendiculars are all equal and each of them is a radius of the excribed circle. Also the two tangents from each angle to the excribed circle are equal; and therefore Ba' is equal to Bc', Ca' to Cb', and Ab' to Ac'. Hence Ab' and Ac' are together equal to the perimeter of the triangle and each of them to the semi-perimeter. But because the two triangles AcO and $Ac'O'$ are similar therefore $Ac : Oc :: Ac' : O'c'$ and (Euclid VI. 16) the rectangle Ac, $O'c'$ is equal to the rectangle Oc, Ac', which by Prop. I. is the area of the triangle.

Numerically, if the radius of the circle excribed on the side BC be represented by α, this may be written

$$(s - a)\alpha = rs = \text{the area.}$$

PROP. III†. *A triangle is a mean proportional to the rectangle contained by the semi-perimeter and its excess over one of the sides, and the rectangle contained by the excess of the semi-perimeter over each of the other sides.*

With the same figure as before the right-angled triangles BOc and $BO'c'$ have the angles BOc and OBc equal to the angles $O'Bc'$ and $BO'c'$ each to each. Therefore these triangles are similar and (Euclid VI. 4) $Bc : cO :: c'O' :: c'B$.

Now, on the first and second of these lines let rectangles of altitude Ac' be constructed, and on the third and fourth rectangles of altitude Ac. Then (Euclid VI. 1) $Ac', Bc : Ac', cO :: Ac, c'O' : Ac, c'B$.

But the rectangle Ac', cO is equal to the triangle ABC by Prop. I. and the rectangle $Ac, c'O'$ is equal to ABC by Prop. II.

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*The proof of this is left to the reader, or he may consult Thomson's *Euclid*, IV. 4.

†This most useful proposition was known to the Greeks of Alexandria, and by them communicated to the Arabians, but seems to have "been reinvented in Europe about the latter part of the 15th century." Leslie's *Geometry*, 1828, V. 19, where the above demonstration (nearly) will be found.

Therefore the triangle is a mean proportional between the rectangles Ac', Bc and Ac, $c'B$, that is between the rectangle contained by the semi-perimeter and its excess over the side CA, and the rectangle contained by its excess over the sides BC and AB respectively.

Writing this numerically, and supposing the area to be represented in square units by the number Δ, it becomes

$$s(s-b) : \Delta :: \Delta : (s-a)(s-c)$$

or

$$\Delta^2 = s(s-a)(s-b)(s-c),$$

whence the area can be calculated in square units when the lengths of the sides are given numerically in units.

Also by Prop. II. $r^2 = \dfrac{(s-a)(s-b)(s-c)}{s}$.

CHAPTER III.

OF SYMBOLS OF QUANTITY.

Angles not limited in magnitude. In Euclid an angle is not defined as a magnitude but as the inclination of two lines, which never exceeds two right angles: and in most of the propositions in Euclid it is not necessary to treat an angle otherwise than as a change of direction of a line. But where an angle is treated as a magnitude (notably in Euclid VI. 33 and consequently in III. 7, on which it depends) any multiple whatever of an angle is termed an angle. So also in Trigonometry, where angular magnitude in general is treated numerically, it is desirable to use the term angle for the sum of a number of angles, which may be greater than two or than any number of right angles. In the same way an arc of a circle may be greater than a circumference or any number of circumferences.

Negative quantities. Again, when magnitudes are represented numerically by algebraic symbols, the values of which are defined but not specified, it is often desirable to express a difference without limiting the generality of the expression by stating which of the symbols stands for the greater number. For instance, if a distance a miles be measured from a fixed point O along (or parallel to) a given line in a standard direction, say east, to A; and a line AB be cut off from OA by measuring from A in the opposite direction, or westward, a distance b miles, the distance OB may be said to be $= (a-b)$ miles east of O not only in the case where $a > b$, but also when $a < b$, if it be agreed to interpret the result as meaning $a-b$ east of O (or in the standard direction) if $a-b$ is a $+$ number, and $b-a$ miles west of O (or in the contrary direction), when $a-b$ is a $-$ number.

Then the standard direction may be called the $+$ (or positive) direction, and the contrary direction the $-$ (or negative) direction.

In the same way, if A be a feet above O and B be b feet below A, B is $a - b$ feet above O if $a > b$, and $b - a$ feet below O if $a < b$. To express the result by one formula we may say that B is $a - b$ feet above O in both cases, if we interpret the $+$ sign as meaning upwards from O and the $-$ sign as meaning downwards (or in the contrary direction) from O.

Thus, in the case of lines measured along (or parallel to) a specified line from a given point (or *origin*) the sign $+$ is conveniently prefixed (or understood) before lines measured in a standard direction and the sign $-$ before those measured in the contrary direction.

Again, with reference to angles, if the hand of a going clock be put back through an angle θ, then, after the time during which the hand moves through an angle ϕ, the hand will make an angle $\theta - \phi$ with its present position, the angle being $+$ and measured in the opposite way to that in which the hand of the clock moves, if $\theta > \phi$; or $-$ and measured in the contrary direction, if $\theta < \phi$.

In what follows, an angle will be considered as if produced by the revolution of a radius of a circle, the direction of revolution from an initiatory position being considered as $-$ or $+$ according as it takes place in the direction of the motion of the hand of a clock or the reverse.

CHAPTER IV.

OF THE UNIT OF ANGULAR MAGNITUDE.

In order to treat angular magnitude numerically it is necessary to use some fixed angle as a standard of comparison, by reference to which the magnitudes of angles under consideration may be denoted.

The angle of easiest construction is the angle of an equilateral triangle, which is also two-thirds of a right angle.

For the purpose of expressing simply fractions of the standard, the sexagesimal division (or division into 60ths) of the standard is probably the most convenient (because the third, fourth, fifth, sixth, tenth, twelfth, fifteenth, twentieth and thirtieth are all exact numbers of sixtieths).

For such reasons perhaps the sexagesimal scale, which has prevailed since the time of Ptolemy*, was originally adopted. It is still employed, and we have the following
Notation for angles in aliquot parts of a right angle: —

The 90th part of a right angle (or the 60th of the angle of an equilateral triangle) is called a degree.

One degree is denoted by 1°; so that a right angle is 90°; the angle of an equilateral triangle is 60°.

The 60th part of a degree is a minute, denoted by $1'$, $\therefore 1^\circ = 60'$.

The 60th part of a minute is a second, denoted by $1''$, $\therefore 1' = 60''$.

Fractions of a second are now usually denoted by decimals, but in older books, as for instance in Newton's *Principia*, the sexagesimal division is carried farther, so that

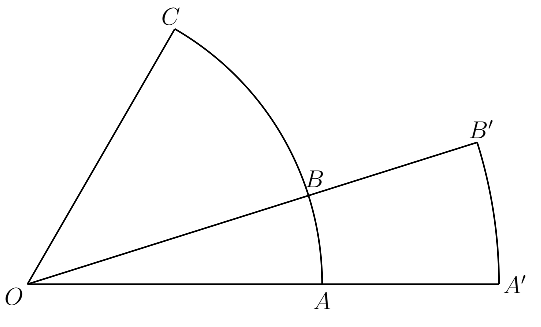
$$1'' = 60''', \quad 1''' = 60^{\text{iv}}, \quad 1^{\text{iv}} = 60^{\text{v}}.$$

*See article "Arithmetic" in the *Encyclopædia Metropolitana*, p. 401, § 39.

This notation is used for many practical purposes*.

Circular Measure. In formulæ, involving explicitly the numerical value of an angle, it is more suitable and it is usual to represent the angle by its ratio to the angle subtended at the centre of a circle by an arc equal to the radius. This angle (Chap. I. Prop. I. Cor. 2) is invariable, that is, is the same whatever radius be taken, and can therefore be used with propriety for a standard of comparison. It is called the *unit of circular measure*, and the ratio of any angle to this unit is called the *circular measure* of the angle.

The circular measure of an angle is also the ratio of the arc subtending the angle at the centre of any circle to the radius. For let AB be the arc subtending the angle AOB, of which the circular measure



is θ, at the centre of the circle of which the radius $OA = R$. And let AOC be subtended by the arc $AC = R$. Then AOC is the unit; and (Euclid VI. 33) $AOB : AOC :: AB : AC$,

$$\text{or } \theta = \frac{AB}{AC} = \frac{AB}{R},$$

$$\text{and the arc } AB = R\theta.$$

*The centesimal division of the right angle into 100 grades, &c. proposed at the French Revolution, though adopted in the *Mécanique Céleste* of Laplace, has been abandoned even in France.

The ratio of the arc to the radius is obviously the same whatever radius be taken. For if $A'B'$ be the arc subtending the same angle at the centre of the circle of radius $OA' = R'$, we have $AB : A'B' :: R : R'$,

$$\text{or } \frac{AB}{R} = \frac{A'B'}{R'}.$$

The circular measure of a right angle

$$= \frac{\frac{1}{4}\text{ circumference}}{R} = \frac{\pi}{2}.$$

And the number of degrees in the unit of circular measure

$$= \frac{180^\circ}{\pi}.$$

Complementary angles. Two angles are said to be complements, each of the other, when their sum is a right angle. Hence the complement of an angle A° contains $90^\circ - A^\circ$, and the circular measure of the complement of θ is $\dfrac{\pi}{2} - \theta$. If $A^\circ > 90^\circ$, or $\theta > \dfrac{\pi}{2}$, the complement is $-$ and to be measured in the negative direction.

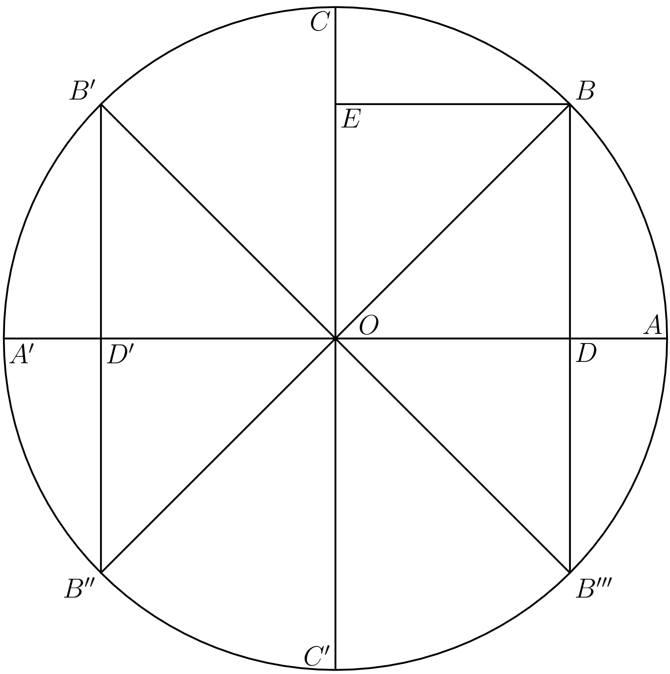
Supplementary angles. Two angles are supplements, each of the other, when their sum is two right angles. Hence the supplement of A° is $180^\circ - A^\circ$; and the circular measure of the supplement of θ is $\pi - \theta$. If $A^\circ > 180^\circ$, or $\theta > \pi$, the supplement is $-$ and to be measured in the negative direction.

CHAPTER V.

CIRCULAR FUNCTIONS, OR TRIGONOMETRICAL RATIOS.

Function. When one magnitude or ratio is so connected with another that the former changes with the latter, but is determinable for any given value of the latter, the former is said to be a function of the latter.

Hence certain ratios, which depend on the value of the angle, or its circular measure, are called circular functions.



Let $ACA'C'$ be any circle, O its centre, $A'OA$, COC' two diameters at right angles dividing the circle into four quadrants.

Let OA, OC be the + directions for lines measured from O along these lines respectively. Let OA be taken as the initial line for angles, and ABC the + direction for angles measured from OA. Let R be the radius of the circle, and $\theta \left(\frac{AB}{R}\right)$ be the circular measure of AOB.

1. *Sin θ.* The sine of the angle AOB is the ratio to the radius of the perpendicular from the end of the arc subtending the angle on the initial line OA, $\therefore \sin\theta = \frac{BD}{R}$.

If the position of B be above the line AOA', $\sin\theta$ is +; if below, $\sin\theta$ is −.

2. *Cos θ.* The cosine of an angle is the sine of its complement, or

$$\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right).$$

Conversely

$$\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right).$$

The angle AOB being θ, and $COA = \frac{\pi}{2}$,

$$COB = \frac{\pi}{2} - \theta;$$

therefore

$$\cos\theta = \sin COB = \frac{BE}{R} = \frac{OD}{R}.$$

Hence, if OD is measured towards A, or if B lie to that side of COC', $\cos\theta$ is +; if OD is measured towards A', or if B lies on the same side of COC' as A', $\cos\theta$ is −.

Variation of the Sine and Cosine. We may examine how the values of these two functions change with the variation of the angle.

When AB is small, BD is small and $+$; $OD =$ the radius nearly and $+$. As θ increases from 0 to $\frac{\pi}{2}$, BD and OD remain both $+$, but BD increases from 0 to R, and OD decreases from R to 0; therefore

$$\begin{gathered}\sin\theta \text{ increases from } +0 \text{ to } +1;\\ \cos\theta \text{ decreases from } +1 \text{ to } 0.\end{gathered}$$

As θ increases from $\frac{\pi}{2}$ to π, BD remains $+$, but diminishes from $+R$ to 0; also OD becomes $-$, and $-OD$ increases from 0 to $+R$; therefore

$$\begin{gathered}+\sin\theta \text{ decreases from } +1 \text{ to } 0;\\ -\cos\theta \text{ increases from } 0 \text{ to } +1.\end{gathered}$$

As θ increases from π to $\frac{3\pi}{2}$, BD becomes $-$, and $-BD$ increases from 0 to $+R$; also OD is $-$, and $-OD$ decreases to 0; so that

$$\begin{gathered}-\sin\theta \text{ increases from } 0 \text{ to } +1;\\ -\cos\theta \text{ decreases from } +1 \text{ to } 0.\end{gathered}$$

As θ increases from $\frac{3\pi}{2}$ to 2π, $-BD$ decreases to 0; and $+OD$ increases to $+R$, so that

$$\begin{gathered}-\sin\theta \text{ diminishes from } +1 \text{ to } 0;\\ +\cos\theta \text{ increases from } 0 \text{ to } +1.\end{gathered}$$

After this the values from 2π to 4π are the same as from 0 to 2π: and if m be any whole number

$$\begin{gathered}\sin\theta = \sin(2m\pi + \theta);\\ \cos\theta = \cos(2m\pi + \theta).\end{gathered}$$

If equal angles, $+\theta$ and $-\theta$, be measured in opposite directions from OA, and the other ends of the arcs subtending them be joined by the straight line BB''', AOA will bisect BB''' at right angles in the point D.

Therefore the sines of these two angles will be equal but of opposite sign, and their cosines will be equal in sign as well as magnitude; or

$$\sin(-\theta) = -\sin\theta;$$
$$\cos(-\theta) = +\cos\theta.$$

If $A'B'$, $A'B''$ be arcs, each $= AB = R\theta$, but measured from A'; the circular measure of the angles subtended by AB', AB'' are respectively $\pi - \theta$, $\pi + \theta$; and the perpendicular $B'D' = +BD$, $B''D = -BD$, and $OD' = -OD$. Consequently

$$\sin(\pi-\theta) = \sin\theta, \qquad \cos(\pi-\theta) = -\cos\theta;$$
$$\sin(\pi+\theta) = -\sin\theta, \qquad \cos(\pi+\theta) = -\cos\theta.$$

And, since both sine and cosine remain unchanged when the angle is increased by a multiple of 2π (say by $2m\pi$), we have

$$\begin{aligned}
&\sin(2m\pi+\theta) = +\sin\theta;\\
&\sin(-\theta) = \sin(2m\pi-\theta) = -\sin\theta;\\
&\sin(\pi-\theta) = \sin\bigl((2m+1)\cdot\pi-\theta\bigr) = +\sin\theta;\\
&\sin(\pi+\theta) = \sin\bigl((2m+1)\cdot\pi+\theta\bigr) = -\sin\theta.
\end{aligned}$$

And

$$\begin{aligned}
&\cos(2m\pi+\theta) = +\cos\theta;\\
&\cos(-\theta) = \cos(2m\pi-\theta) = +\cos\theta;\\
&\cos(\pi-\theta) = \cos\bigl((2m+1)\cdot\pi-\theta\bigr) = -\cos\theta;\\
&\cos(\pi+\theta) = \cos\bigl((2m+1)\cdot\pi+\theta\bigr) = -\cos\theta.
\end{aligned}$$

By these the sine and cosine of an angle of any magnitude can be obtained from the sine or cosine of an acute angle.

It should be observed that, while the number θ continuously increases, the numbers $\sin\theta$, $\cos\theta$ pass through a series of values between $+1$ and -1, and return to the same values again for every increase of 2π in the value of θ. They are therefore said to be *periodic functions* of θ, of which the period is 2π.

Since the perpendicular from the centre of a circle on any chord bisects it at right angles, the ratio of the chord to the radius is twice the sine of half the angle subtended by the chord.

Hence, if we can calculate the ratio to the radius of the side of an inscribed polygon, the sine of half the angle subtended by the side is at once known.

For instance, the side of a square inscribed in a circle $= R\sqrt{2}$, and it subtends an angle of $\dfrac{\pi}{2}$ or $90°$. Therefore

$$\sin\frac{\pi}{4} = \sin 45° = \frac{1}{2}\sqrt{2} = \cos 45° = \cos\frac{\pi}{4}.$$

The side of the hexagon inscribed is $= R$, and it subtends $\dfrac{\pi}{3}$ or $60°$. Therefore

$$\sin\frac{\pi}{6} = \sin 30° = \frac{1}{2} = \cos 60° = \cos\frac{\pi}{3}.$$

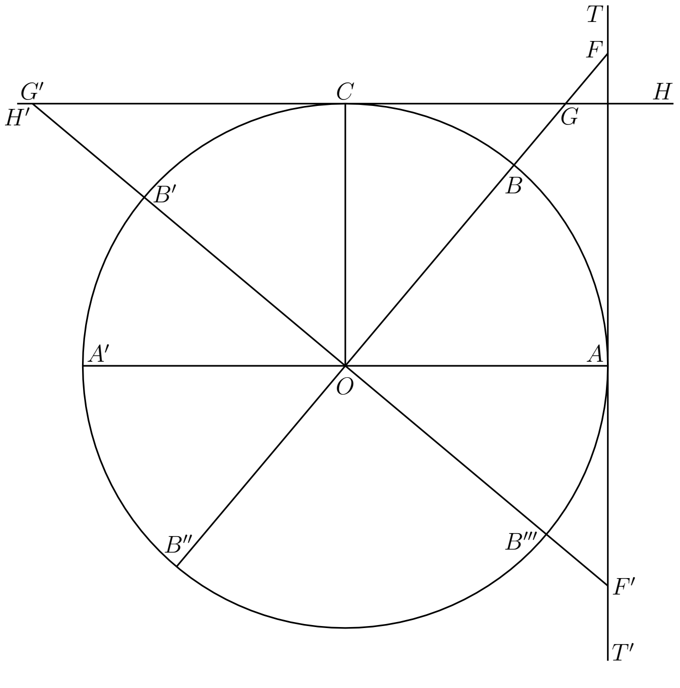
The side of the inscribed equilateral triangle $= R\sqrt{3}$, and it subtends $\dfrac{2\pi}{3}$ or $120°$. Hence

$$\sin\frac{\pi}{3} = \sin 60° = \frac{1}{2}\sqrt{3} = \cos 30° = \cos\frac{\pi}{6}.$$

The side of the regular decagon inscribed is (Euc. IV. 10, and II. 11) $\dfrac{\sqrt{5}-1}{2}R$, and it subtends an angle $\dfrac{\pi}{5}$ or $36°$. Hence

$$\sin\frac{\pi}{10} = \sin 18° = \frac{\sqrt{5}-1}{4} = \cos\frac{4\pi}{10} = \cos 72°.$$

3. *Tan θ.* The tangent of an angle is the ratio to the radius of the part of the line, touching at the initial end the arc subtending the



angle, intercepted between the point of contact and the radius produced through the other end of that arc. In the diagram

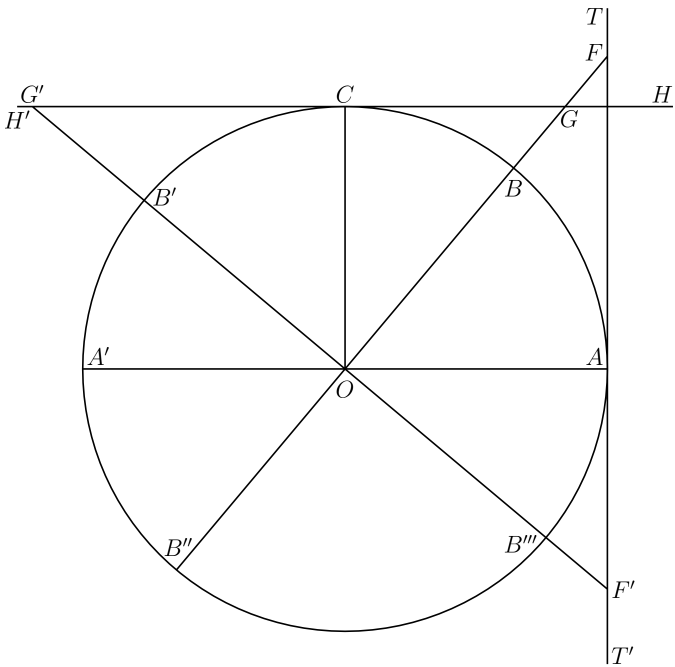
$$\tan\theta = \frac{AF}{R} \text{ and } AF = R\tan\theta.$$

If AF be measured towards T, it is to be considered $+$, and $-$, if in the contrary direction towards T'.

4. *Cot* θ. The cotangent of an angle is the tangent of the complement of the angle. Whence

$$\cot\theta = \tan\left(\frac{\pi}{2} - \theta\right) \text{ and } \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right).$$

The cotangent may hence be geometrically defined as follows. Let OC be drawn perpendicular to OA. Then COB is the complement of AOB, or $COB = \dfrac{\pi}{2} - \theta$. If AOB be greater than AOC, or $\theta > \dfrac{\pi}{2}$, COB will be measured from CO in the other direction, and $\dfrac{\pi}{2} - \theta$ will be $-$ to correspond.



Draw $H'CH$ touching the arc subtending the complement of θ at the point C, which is always an extremity of that arc, and through the other extremity of the arc produce the radius to meet it; the tangent of the complement, or cotangent, is the ratio to the radius of the part of HCH' intercepted. In the diagram

$$\cot\theta = \frac{CG}{R} \text{ and } CG = R\cot\theta.$$

If CG is measured in the direction from C to H' the cotangent is $-$.

Variation of the Tangent and Cotangent. In the first quadrant, when the angle is indefinitely small, the tangent is indefinitely small, and the cotangent indefinitely great; since AF diminishes, and CG increases without limit, as AOB decreases. Hence it is usually said that $\tan 0 = 0$, and $\cot 0 = \infty$.

As the angle increases to $90°$, AF increases and CG decreases, both drawn in the $+$ direction; so that $\tan\theta$ increases and $\cot\theta$ decreases, as θ increases from 0 to $\frac{\pi}{2}$, and when $\theta = \frac{\pi}{2}$, $\tan\theta = \infty$, $\cot\theta = 0$.

In the second quadrant, where θ increases from $\frac{\pi}{2}$ to π, AF' is measured in the $-$ direction AT', and CG' in the $-$ direction CH' for an angle AOB': so that $\tan\theta$ and $\cot\theta$ are both $-$.

Also, AF' decreases from an indefinitely great distance in the direction AT', when θ barely exceeds $\frac{\pi}{2}$, to 0 when $\theta = \pi$; while CG' increases from 0 to an indefinitely great distance in the direction CH'. Hence, as θ increases from $\frac{\pi}{2}$ to π, $-\tan\theta$ decreases from ∞ to 0 and $-\cot\theta$ increases from 0 to ∞.

In the third quadrant, or when θ increases from π to $\frac{3\pi}{2}$, the lines AF and CG are again measured in the positive direction, and the tangent and cotangent are therefore both +, and vary in magnitude as in the first quadrant.

In the fourth quadrant, where θ is greater than $\frac{3\pi}{2}$, and less than 2π, the lines are again drawn in the $-$ direction, and the tangent and cotangent vary just as in the second quadrant.

If two equal angles, as AOB and AOB''', or AOB' and AOB'', be measured in opposite directions from OA, it is obvious that their tangents and cotangents are equal in magnitude, but opposite in sign; or

$$\tan(-\theta) = -\tan\theta; \text{ and } \cot(-\theta) = -\cot\theta.$$

Also if $A'OB'$ and AOB are equal angles, so that AOB' and AOB are supplementary, their tangents and cotangents are equal, but of opposite signs; that is

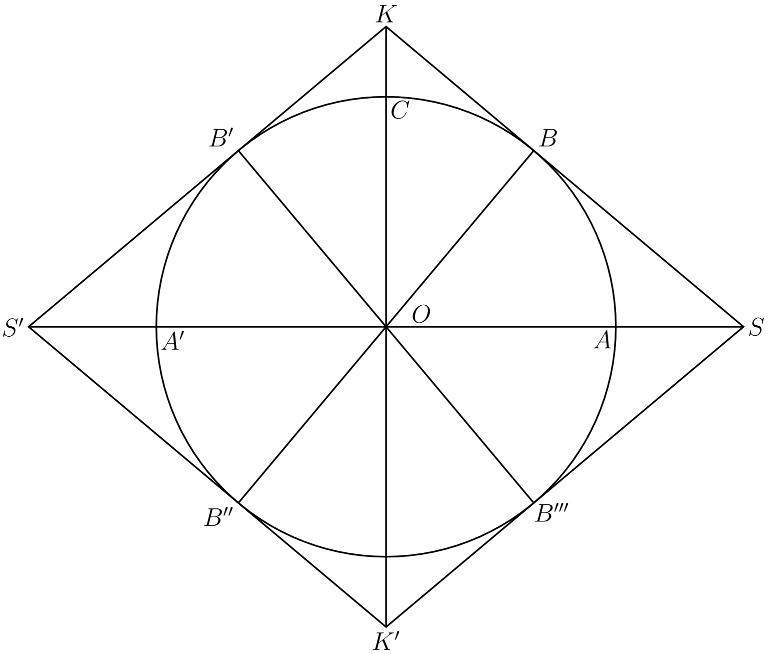
$$\tan(\pi-\theta) = -\tan\theta; \quad \cot(\pi-\theta) = -\cot\theta.$$

If m be any whole number, it is clear that the angle $2m\pi+\theta$ begins and terminates at the same point as the angle θ. Therefore,

$$\begin{aligned}
\tan(2m\pi+\theta) &= \tan\theta; & \cot(2m\pi+\theta) &= \cot\theta;\\
\tan(2m\pi-\theta) &= -\tan\theta; & \cot(2m\pi-\theta) &= -\cot\theta;\\
\tan\bigl((2m+1)\cdot\pi-\theta\bigr) &= -\tan\theta; & \cot\bigl((2m+1)\cdot\pi-\theta\bigr) &= -\cot\theta;\\
\tan\bigl((2m+1)\cdot\pi+\theta\bigr) &= \tan\theta; & \cot\bigl((2m+1)\cdot\pi+\theta\bigr) &= \cot\theta.
\end{aligned}$$

Whence the tangent and cotangent of any angle can be found from those of an acute angle.

5. *Sec θ.* The secant of an angle is the ratio to the radius of the initial radius produced to meet the tangent at the other end of the arc subtending the angle.



6. *Cosec θ.* The cosecant is the secant of the complement.

In the diagram, AOB being the angle, and COB the complement, KS the tangent at B;

$$\sec\theta = \frac{OS}{R}; \quad \operatorname{cosec}\theta = \frac{OK}{R};$$
$$OS = R\sec\theta; \quad OK = R\operatorname{cosec}\theta.$$

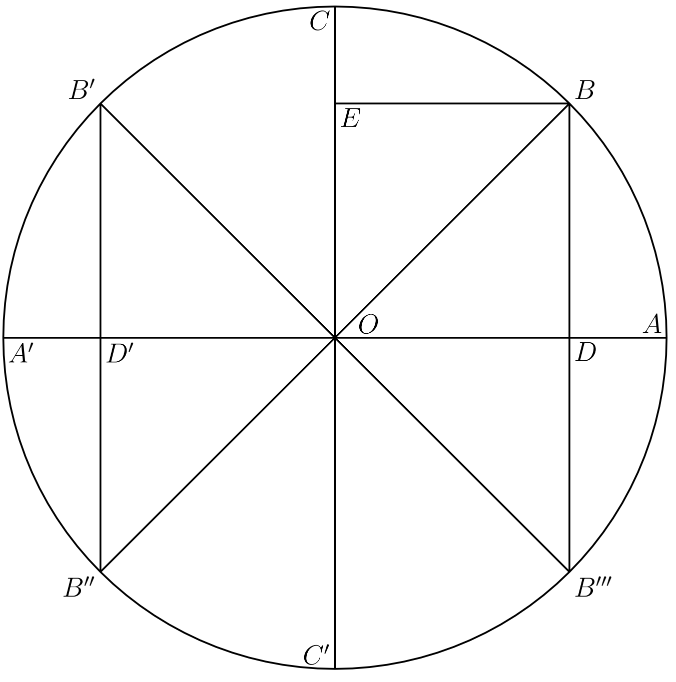
Then $\sec 0 = 1$; and the secant increases till $\sec\dfrac{\pi}{2} = \infty$. When θ is between $\dfrac{\pi}{2}$ and $\dfrac{3\pi}{2}$, $\sec\theta$ is $-$, and $-\sec\theta$ decreases from ∞ (when $\theta = \dfrac{\pi}{2}$) till $-\sec\pi = 1$, and then increases again till $-\sec\dfrac{3\pi}{2} = \infty$;

after which, while θ increases from $\dfrac{3\pi}{2}$ to 2π, $+\sec\theta$ decreases from ∞ to 1.

Similarly $\operatorname{cosec}\theta$ is $+$ in the first and second quadrants diminishing from $\operatorname{cosec} 0 = \infty$ till $\operatorname{cosec}\dfrac{\pi}{2} = +1$ and then increasing till $\operatorname{cosec}\pi = +\infty$.

In the third and fourth quadrants $\operatorname{cosec}\theta$ is $-$, and $-\operatorname{cosec}\theta$ decreases from ∞ at the beginning of the third quadrant till $-\operatorname{cosec}\dfrac{3\pi}{2} = 1$, and then increases till $-\operatorname{cosec} 2\pi = \infty$.

7. *Versin θ.* The versed sine of an angle is the ratio to the radius of



the part of the initial radius (produced if necessary) which is intercepted between its extremity A and the perpendicular from the other end B of the arc subtending the angle. In the diagram, we have

$$\operatorname{versin}\theta = \frac{AD}{R}; \text{ and } AD = R \operatorname{versin}\theta.$$

The line AD is always drawn in one direction and $\operatorname{versin}\theta$ is always $+$.

As θ increases from 0 to π, $\operatorname{versin}\theta$ increases from 0 to 2; and as θ increases from π to 2π, $\operatorname{versin}\theta$ diminishes from 2 to 0.

The seven ratios defined above are altogether independent of the size of the circle described, and depend only on the angle. They are therefore *functions* of the circular measure θ, remaining the same for the same value of θ whatever radius be taken, but changing with the value of θ; and they are numbers calculable from the number θ. The lines, to which their ratios correspond, depend partly on the value of θ and partly on the radius, and are expressed by the radius multiplied by the corresponding circular function.

Thus if the angle $AOB = \theta$ in the diagrams of this Chapter,

$$\begin{gathered} BD = OB\sin\theta = OD\tan\theta, \\ EB = OB\cos\theta = EO\cot\theta, \\ AF = OA\tan\theta = OF\sin\theta, \\ CG = OC\cot\theta = OG\cos\theta, \\ OS = OB\sec\theta, \quad OK = OB\operatorname{cosec}\theta. \end{gathered}$$

Relations between the circular functions of the same angle. If the angle AOB and the radius of the circle be the same in these diagrams, the triangles BOD, FAO, SOB are right-angled and similar. Hence expressing their sides in terms of the radius and the circular function corresponding to each, we have (Euc. I. 47)

$$R^2 = R^2 \sin^2\theta + R^2 \cos^2\theta,$$

$$\text{or } \sin^2\theta + \cos^2\theta = 1. \tag{1}$$

$$R^2 \sec^2\theta = R^2 + R^2 \tan^2\theta,$$

$$\text{or } \sec^2\theta = 1 + \tan^2\theta. \tag{2}$$

And putting $\dfrac{\pi}{2} - \theta$ for θ in this

$$\operatorname{cosec}^2\theta = 1 + \cot^2\theta. \tag{3}$$

Also (Euc. VI. 4) $R\tan\theta : R :: R\sin\theta : R\cos\theta$,

$$\text{or } \tan\theta = \frac{\sin\theta}{\cos\theta} \tag{4}$$

$$R\sec\theta : R :: R : R\cos\theta,$$

$$\text{or } \sec\theta = \frac{1}{\cos\theta}. \tag{5}$$

From which putting $\dfrac{\pi}{2} - \theta$ for θ,

$$\operatorname{cotan}\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} \tag{6}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} \tag{7}$$

And $R\operatorname{versin}\theta = R - R\cos\theta$

$$\text{whence } \operatorname{versin}\theta = 1 - \cos\theta. \tag{8}$$

From these equations all the functions can be found, when one has been given. For instance, from the values of the sine and cosine given above (p. 25) we get

$$\begin{aligned}
\tan 45^\circ &= \cot 45^\circ = 1,\\
\sec 45^\circ &= \operatorname{cosec} 45^\circ = \sqrt{2},\\
\tan 30^\circ &= \operatorname{cotan} 60^\circ = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3},\\
\sec 30^\circ &= \operatorname{cosec} 60^\circ = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3},\\
\cot 30^\circ &= \tan 60^\circ = \sqrt{3} = \sqrt{3},\\
\operatorname{cosec} 30^\circ &= \sec 60^\circ = 2,\\
\cos 18^\circ &= \frac{1}{4}\sqrt{10 + 2\sqrt{5}},\ \&c.
\end{aligned}$$

CHAPTER VI.

OF LOGARITHMIC TABLES.

Tables have been formed of the values of the circular functions of angles at intervals of $1'$, or in more minute tables at intervals of $10''$, from $0°$ to $45°$. It is unnecessary to calculate those of greater angles, as the angles between $45°$ and $90°$ are complements of the angles between $0°$ and $45°$. Methods of calculation are given in Higher Trigonometry. Such tables are called Tables of Natural Sines, Cosines, &c. For practical calculations they are superseded by logarithmic tables, the use of which may be shortly explained.

Logarithms of ordinary numbers may be defined to be numbers, so calculated from the ordinary numbers, that the sum of the logarithms of two numbers is the logarithm of their product. Thus, if m and n be two numbers,

$$\log m + \log n = \log(m \times n). \tag{1}$$

Now

$$\begin{gathered}\log(m \div n) + \log n = \log n(m \div n) = \log m;\\ \therefore \log m - \log n = \log(m \div n). \qquad (2)\end{gathered}$$

From (2) it follows, putting $m = n$, that

$$\log 1 = 0, \tag{3}$$

and that the logarithm of a proper fraction is $-$.

Also from (1)

$$\begin{aligned}\log n^2 &= 2\log n,\\ \log n^3 &= 3\log n, \quad \&c.\end{aligned}$$

and, generally, $\log n^p = p \log n$, if p is an integer,
and therefore $q \log n^{\frac{p}{q}} = p \log n$,

$$\text{or } \log n^{\frac{p}{q}} = \frac{p}{q} \times \log n.$$

So that whether x is an integer, or a fraction,

$$\log n^x = x \log n. \tag{4}$$

If $a =$ the number of which the logarithm is 1, a is said to be the base of the system of logarithms: then denoting logarithms in the system of which the base is a by $\log_a$, (4) becomes

$$\log_a n = x, \text{ when } a^x = n. \tag{5}$$

Common logarithms are those for which the base is 10.

Hence in (3) and (5) denoting common logarithms by log; $\log 1 = 0$; $\log 10 = 1$; $\log 100 = 2$; $\log 1000 = 3$; and so on.

Equation (5) shews that the logarithm increases with the number: therefore it appears that the common logarithm of a number of one integral digit is a proper fraction; that of a number of 2 digits is 1 + a fraction; of 3 digits 2 + a fraction, and so on. The integral part, or *characteristic*, of the logarithm, therefore, is one less than the number of integral digits and is known by inspection, so that it is usual in the tables to give only the decimal part of the logarithm, or the *mantissa*.

Since also by (1) and (2),

$$\log(10^m \times n) = m + \log n,$$
$$\text{and } \log(n \div 10^m) = -m + \log n,$$

it appears that multiplication or division by any power of 10 will affect only the characteristic of the logarithm, and that the decimal part of the logarithm, or mantissa, will be the same for numbers consisting of the same succession of digits followed or preceded by any number of ciphers, and with a decimal point occurring between any two digits.

It will be seen that all the arithmetical operations for calculation are simplified by the use of logarithmic tables.

Thus, with tables, the result of multiplication is calculated by adding logarithms, of division by subtracting logarithms, of raising to a power by multiplying a logarithm by a number, of extracting a root by dividing a logarithm by a number.

Calculations are thus so much abbreviated* that it is very desirable in establishing formulæ for the calculation of some of the sides and angles of triangles from given values of others, that the formulæ should be adapted to logarithmic calculation, as is done in the next chapter.

It is also necessary to have logarithmic tables of the Trigonometrical ratios.

Since the sine of an angle is always less than 1, the common logarithm of the sine is a $-$ number. The same is true of the cosine, of the tangent of an angle $< 45°$, &c. To avoid the use of negative numbers, the tabular logarithms of the circular functions are the common logarithms of these ratios increased by 10: which must be remembered in using the tables in calculations.

Where, as is generally the case in practice, the logarithm of a given number, or, conversely, the number for a given logarithm is not actually to be found in the tables, these can be found approximately from the

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*Laplace has remarked with reference to the value of the invention, that "by reducing to a few hours what otherwise is the labour of several months, it doubles, so to speak, the life of the astronomer, and spares him the errors and annoyance inseparable from long calculations."

principle that, for considerable numbers of small difference, the difference of two numbers is nearly proportional to the difference of their logarithms. Whence the difference between a number and the nearest number in the tables being known, the difference to be added to the nearest logarithm in the tables can be approximately found, and the converse.

The same method applies also to the tabular logarithms of the circular functions, it being generally approximately true that the difference of two tabular logarithms of a circular function is proportional to the difference of the corresponding angles, when that difference is small, and the angles are neither very small nor very near 90°*.

*Logarithms were invented by John Napier, who was born in 1550 and died 4th April, 1617. He was descended from and claimed to represent the elder branch of the Earls of Lennox. From him are descended the Baronet of Milliken and the Lord Napier. But the inventor of Logarithms was not a Peer, and should not be styled Baron Napier as is often done. The mistake arises from his having been "Baron" in the sense of proprietor of the Barony of Merchistoun, near Edinburgh. In a paper dated 23rd April, 1584, at Gartness on the Endrick (where were his Lennox Estates and where it is a local tradition that he calculated his Canon of Logarithms), he signs himself "Jhone Neper *Fear* of Merchiston." The word fear is the Celtic equivalent of Baron.

His "Canon," or table of Logarithms, was published in 1614.

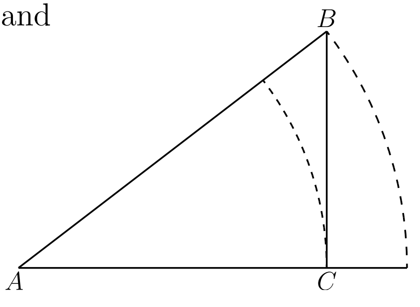
CHAPTER VII.

SOLUTION OF TRIANGLES.

A TRIANGLE is said to be solved when the sides and angles are calculated from the data.

Right-Angled Triangles. Let ABC be a triangle. Let $C = 90°$; and let $BC = a$, $CA = b$, $AB = c$.

Then if the side AB be taken as radius, the ratio of BC to AB is the sine of A; and of AC to AB is its cosine; or



$$a = c \sin A; \quad b = c \cos A.$$

Similarly

$$b = c \sin B; \quad a = c \cos B.$$

If AC be taken as radius, since CB touches the arc described, the ratio of CB to CA is the tangent, and of AB to CA is secant of A; so that

$$a = b \tan A; \quad c = b \sec A.$$

And similarly

$$b = a \tan B; \quad c = a \sec B.$$

From these equations, it is easy logarithmically to solve a right-angled triangle when a side and an angle, or when two sides are given.

Given c and A, we have $B = 90° - A$,

$$\log a = \log c + \text{tab} \log \sin A - 10,$$
$$\log b = \log c + \text{tab} \log \cos A - 10;$$

whence B, a, b are found.

Given a and A, we have $B = 90° - A$,

$$\log b = \log a + \text{tab} \log \tan B - 10,$$
$$\log c = \log a + \text{tab} \log \sec B - 10;$$

whence B, c, b are found.

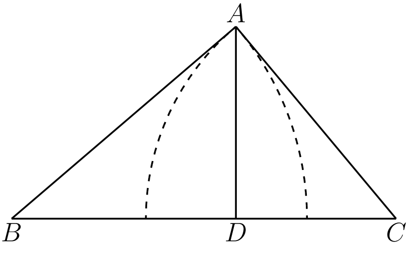
Given c and a, we have

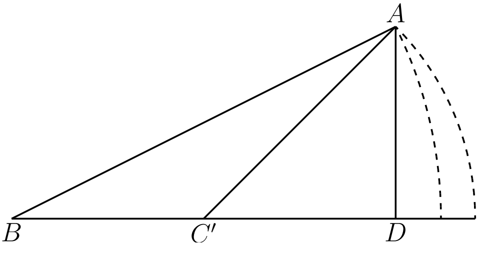
$$\text{tab} \log \sin A = 10 + \log a - \log c,$$
$$B = 90° - A,$$
$$\log b = \log c + \text{tab} \log \cos A - 10;$$

whence A, B, b are found.

Triangles not Right-Angled.

Case I. Let ABC be an acute-angled triangle, ABC' a triangle





having an obtuse angle at C'. In both triangles draw AD perpendicular to BC produced if necessary. Then in both triangles, if AB be taken as radius,

$$AD = AB \sin B = c \sin B;$$

and if CA be taken as radius,

$$AD = CA \sin C = b \sin C.$$

Hence

$$b \sin C = c \sin B,$$

$$\text{or } \frac{b}{\sin B} = \frac{c}{\sin C}.$$

If the perpendicular had been drawn from B to CA we should similarly have got

$$\frac{a}{\sin A} = \frac{c}{\sin C},$$

$$\text{so that } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

or *the sides are proportional to the sines of the opposite angles.* This is true of all triangles. Hence we can logarithmically solve a triangle when a side and the opposite angle are given, and likewise another side or another angle.

Given C and c, and b.

$$\text{Tab} \log \sin B = (\text{tab} \log \sin C - \log c) + \log b,$$

a logarithmic equation for the determination of B.

There is but one acute angle which can have its sine of a particular value: so that, if we know B to be acute, there is but one value of B to be taken. And if $b < c$ the angle B must be acute; and we find it by means of the equation.

Then

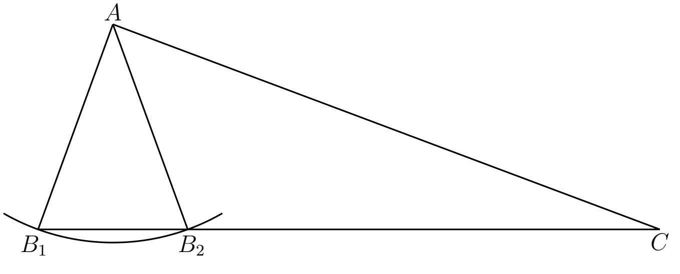
$$A = 180^\circ - (B + C),$$

and

$$\log a = \operatorname{tab} \log \sin A - (\operatorname{tab} \log \sin C - \log c),$$

so that the triangle is solved.

If however $b > c$, we do not know that B is acute, and as the sine of the supplement of an acute angle is the same as the sine of the angle, there are two values of B, the one less than 90°, the other its supplement, which satisfy the above logarithmic relation. That is, there is an *ambiguity* as to the value of B. There will always be this ambiguity in determining the angle of a triangle from its sine, unless there be something to point out whether the angle is acute or obtuse: and therefore it is generally inconvenient to use a method involving the determination of the angle from its sine. But in this instance there is a real *geometrical* ambiguity. For if we construct a triangle with its angle $ACB = C$, its side $CA = b$, and the side opposite $C = c$, it is evident that, if c be $< b$, there are two triangles AB_1C and AB_2C corresponding to the data. The angle opposite CA in the one of these is acute $= B_1$; in the other it is obtuse $= B_2$. But since $AB_1 = AB_2$, $B_2 = 180^\circ - B_1$. In this case therefore, *where two sides and the angle*



opposite the lesser are given, which is called the *ambiguous case*, there are two triangles. The acute angle B_1 can be found from the formula

$$\operatorname{tab} \log \sin B_1 = \log b + (\operatorname{tab} \log \sin C - \log c),$$

and then $B_2 = 180° - B$.

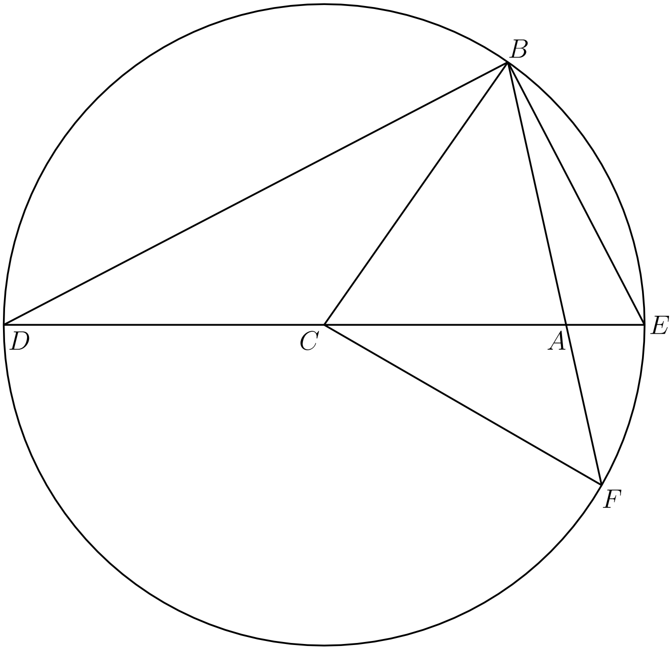
If A_1 and A_2 are the corresponding values of the third angle a_1, a_2 of the third side,

$$A_1 = 180° - (B_1 + C); \quad A_2 = 180° - (B_2 + C),$$

and

$$\begin{aligned}\log a_1 &= \operatorname{tab}\log\sin A_1 - (\operatorname{tab}\log\sin C - \log c),\\ \log a_2 &= \operatorname{tab}\log\sin A_2 - (\operatorname{tab}\log\sin C - \log c).\end{aligned}$$

Case II. Let ABC be a triangle: and let CB be greater than CA.



From C as centre with CB as radius, describe the circle DBE. Produce BA to meet its circumference in F, and CA both ways to meet it in E and D.

Join FC, BD, BE, BF.

Then DBE is a right angle; and

$$DCB = A + B = 2BED;$$

also

$$A = ECF + CFA = ECF + B,$$

and

$$ECF = A - B = 2FBE.$$

So that $BED = \frac{1}{2}(A + B)$; $FBE = \frac{1}{2}(A - B)$.

And BDA is the complement of BED, ABD of FBE.

Now in the triangle DBA,

$$\frac{DA}{\sin ABD} = \frac{BA}{\sin BDA}, \quad \text{or} \quad \frac{a+b}{\cos\frac{1}{2}(A-B)} = \frac{c}{\cos\frac{1}{2}(A+B)}.$$

And in the triangle BAE,

$$\frac{EA}{\sin FBE} = \frac{AB}{\sin BED}, \quad \text{or} \quad \frac{a-b}{\sin\frac{1}{2}(A-B)} = \frac{c}{\sin\frac{1}{2}(A+B)}.$$

By combining these and remembering that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ we get

$$\frac{\tan\frac{1}{2}(A-B)}{\tan\frac{1}{2}(A+B)} = \frac{a-b}{a+b}.$$

These relations give the simplest logarithmic solution of a triangle, when two sides and the contained angle are given*.

Given a, b, and C, we have

$$A + B = 180^\circ - C, \text{ so that } \frac{1}{2}(A+B) \text{ is known.}$$

Then

$$\text{tab} \log \tan \frac{1}{2}(A-B) = \text{tab} \log \tan \frac{1}{2}(A+B) + \log(a-b) - \log(a+b),$$

which determines the acute angle $\frac{1}{2}(A-B)$.

Whence

$$A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B)$$

and

$$B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B)$$

are found.

The third side c could now be found from the relation $\log c = \text{tab} \log \sin C - (\text{tab} \log \sin A - \log a)$, but it is shorter in practice to determine c either from

$$\log c = \text{tab} \log \cos \frac{1}{2}(A+B) - \text{tab} \log \cos \frac{1}{2}(A-B) + \log(a+b),$$

*These formulæ are given in p. 13 of Thacker's *Miscellany*, 1743. Their application to the solution of triangles was pointed out by Professor Wallace, of Edinburgh, in the *Transactions, R.S.E.*, 1823.

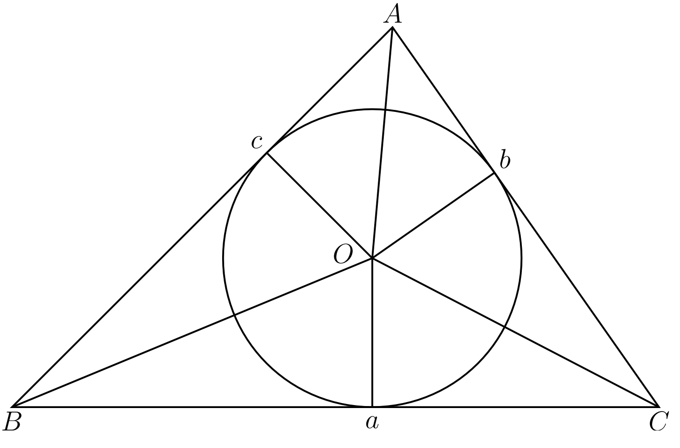
or from

$$\log c = \text{tab}\log\sin\frac{1}{2}(A+B) - \text{tab}\log\sin\frac{1}{2}(A-B) + \log(a-b),$$

Thus when two unequal sides and the angle between them are given, the other side and other angles are found.

If an angle and two equal sides containing it are given, the remaining angles are each the complement of half the given angle, and the third side can be found as in Case I.

Case III. Let ABC be the triangle, O the centre of the inscribed circle, so that AO, BO, CO bisect the angles, and the perpendiculars



Oa, Ob, Oc are each of them a radius of the inscribed circle.

Let this radius $= r$, let the sum of the sides $a+b+c = 2s$; then (Chapter II.)

$$Ab = Ac = s-a, \quad Bc = Ba = s-b, \quad Ca = Cb = s-c.$$

And in the right-angled triangles AOb, BOc, COa, we have

$$r = (s-a)\tan\frac{1}{2}A = (s-b)\tan\frac{1}{2}B = (s-c)\tan\frac{1}{2}C,$$
$$\text{where } r^2 = \frac{(s-a)(s-b)(s-c)}{s}.$$

From these the angles may be found when the sides are given.

Given a, b, c, we get the logarithmic relations

$$\log r = \frac{1}{2}\big\{\log(s-a) + \log(s-b) + \log(s-c) - \log s\big\},$$

and

$$\text{tab}\log\tan\frac{1}{2}A = 10 + \log r - \log(s-a),$$
$$\text{tab}\log\tan\frac{1}{2}B = 10 + \log r - \log(s-b),$$
$$\text{tab}\log\tan\frac{1}{2}C = 10 + \log r - \log(s-c),$$

whence the three angles are easily found. There is also a simple verification of the process afforded by taking the sum of the three angles, which ought to be 180°*.

---

*The formulæ used here were discovered by William Purser of Dublin, in 1632. Consult Wallace's *Geometrical Theorems*.

CHAPTER VIII.

OF TRIGONOMETRICAL SURVEYING.

In the last Chapter methods have been given of calculating the remaining parts of a triangle when one side and two other parts of the triangle are previously known.

In Geodesy, or the application of this part of Trigonometry to surveying, a line, called *the Base*, is measured between two convenient stations; and the angles between lines from these stations to points visible from the stations are measured with appropriate instruments. With these data calculations are made of the distances between new stations, where additional angles may be observed and the operations repeated as often as required. The angles measured by instruments are of three classes.

1. Horizontal angles, or angles in the horizontal plane through the station.

The horizontal angle between two objects is the angle between the vertical planes through the station and each object: it is also called the angle in azimuth.

2. Vertical angles, or angles in a vertical plane through the station.

The angle of elevation (or depression) of an object, also called its altitude, is the vertical angle between the horizontal line and the line from the station to the object.

3. Angles in planes neither horizontal nor vertical.

The angle between two objects is the angle between the lines from the station to each object. This angle is in the plane through the station and the two objects.

In land surveying the angles measured belong to the first or second class, and are measured with a Theodolite or similar instrument.

Angles of the third class are usually measured with a Sextant or similar instrument.

For details of the measurement of a base, and a description of the instruments principally employed in practical surveying, the reader is referred to Professor Rankine's *Manual of Civil Engineering*, Part 1. *Of Engineering Geodesy*, and for the methods employed in a great Trigonometrical survey to the Ordnance Survey *Account of the Principal Triangulation*, published by the Board of Ordnance, 1858. Assuming, however, that a base line and angles can be accurately measured by appropriate instruments, it seems within the scope of this elementary treatise to give instances of the determination of heights and distances from such measurements.

I. The distance between two stations A and B, visible from each other, being ascertained, and the angles between a visible object C and the other station being measured at each station, the distances AC and BC can be found by Case I.

In this instance the angles measured are in the plane through A, B, C.

If the angle of elevation of C be likewise measured at either station, the height of C above that station can be found by solving a right-angled triangle of which the hypotenuse and one of the acute angles are known.

When the horizontal distance of the stations is given and is in a vertical plane through C, only vertical angles are required and the work is simplified; and when in addition one station is in the vertical line through C (which is then said to be "accessible") the measurement of one vertical angle and the solution of one right-angled triangle determine the height of C.

II. From each of two stations A and B, visible from each other, and at a known distance, the angles between the other station and two objects C and D are measured. Then the triangles ABC and ABD can be solved (by Case I.), and the distances AC, BC, AD, BD are found.

If in addition the angles between C and D are measured at either station, the distance CD can be found by Case II.

If the survey extends over so small a fraction of the earth's surface that the vertical lines over the whole may be considered perpendicular to one horizontal plane, and if the distances measured or calculated are the *horizontal distances* of the points (or the "projections" of the actual distances on the horizontal plane), which is the case when the survey is conducted for the purpose of making a plan; then the angles to be measured will all be horizontal angles, and the work will be simpler. And if in addition the angle of elevation of any point be observed from a known horizontal distance, the height of the point above the horizontal plane of reference (which may be "the level of the sea") can be found by the solution of a right-angled triangle.

Where the extent of the earth's surface surveyed is too great for the above supposition, the calculations have to be corrected for the sphericity of the earth (see books on Spherical Trigonometry), and in very great and very accurate surveys also for the earth's deviation from a perfect sphere. (See the *Principal Triangulation* of the Ordnance Survey.)

CHAPTER IX.

OF PROJECTIONS.

THE point where the perpendicular from a given point on a given plane or a given line meets the plane or line is called the projection (or more precisely the orthogonal projection) of the point on the plane or line.

The part of one line intercepted between the projections on it of the extremities of another line, or of a broken line, is called the projection of the second line, or broken line, on the first.

It is convenient to take one direction of the line of projection (say from left to right) as the + direction and the opposite as the − direction; also to consider one end of the projected line as its beginning; and to measure the inclination of the two lines by the angle between the projected line and a line drawn from its beginning parallel to the line of projection and in the + direction.

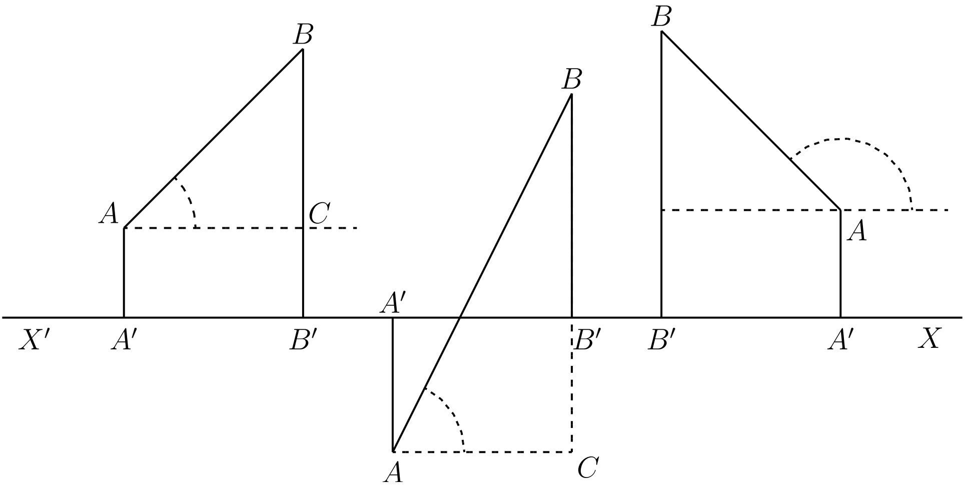
PROP. I. *The projection of a line on a given line is equal to the projected line multiplied by the cosine of the angle between the lines.*

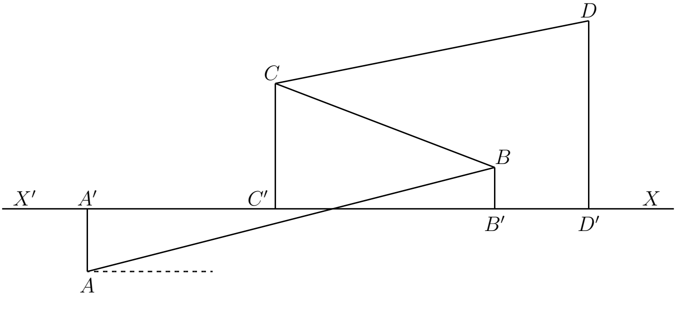
Let AB be the projected line, $X'X$ the line of projection; then in each case, $A'B'$ being the projection, we have

$$A'B' = AB \cos BAC.$$

If BAC be acute, $\cos BAC$ is $+$, and $A'B'$ is $+$ and is measured from A' in the $+$ direction; but if BAC be obtuse, $\cos BAC$ is $-$, and $A'B'$ is $-$ and is measured from A' in the $-$ direction.

PROP. II. *The projection of a broken line is the algebraic sum of the projections of the parts of which it is made up.*





Let $ABCD$ be a broken line, made up of the parts AB, BC, CD. Let $A'B'$, $B'C'$, $C'D'$ be the projections of these parts on $X'X$. Thus $A'D'$ is the projection of the broken line. And

$$A'D' = A'B' + B'C' + C'D'$$

both in the case where all these projections are $+$ and also where, as in the figure, $B'C'$ is $-$.

CHAPTER X.

THE SINE AND COSINE OF THE SUM AND DIFFERENCE OF TWO ANGLES.

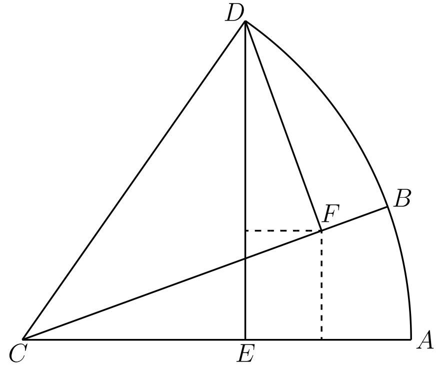
1. *To find the sine and cosine of the sum of two angles.*

Let C be the centre of a circle of which the radius $CA = R$.

Let the circular measure of $ACB = \theta$, and of $BCD = \phi$.

Then $\theta + \phi$ is the circular measure of ACD.

Draw DE, DF perpendicular to CA, CB.



Then

$$DE = R\sin(\theta + \phi); \qquad CE = R\cos(\theta + \phi);$$
$$DF = R\sin\phi; \qquad CF = R\cos\phi.$$

But DE is the projection of the broken line CFD on DE; therefore

$$\begin{aligned} DE &= \text{projection of } CF + \text{projection of } FD \\ &= CF\cos\left(\frac{\pi}{2} - \theta\right) + FD\cos\theta \\ &= CF\sin\theta + FD\cos\theta, \end{aligned}$$
$$\text{i.e. } R\sin(\theta + \phi) = R\cos\phi\sin\theta + R\sin\phi\cos\theta,$$
$$\text{or } \sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi.$$

Again, CE is the projection of the broken line CFD on CA,

$$\begin{aligned}\therefore CE &= \text{projection of } CF + \text{projection of } FD\\ &= CF\cos\theta + FD\cos\left(\frac{\pi}{2}+\theta\right)\\ &= CF\cos\theta - FD\sin\theta;\end{aligned}$$

$$\text{i.e. } R\cos(\theta+\phi) = R\cos\phi\cos\theta - R\sin\phi\sin\theta,$$

$$\text{or } \cos(\theta+\phi) = \cos\theta\cos\phi - \sin\theta\sin\phi.$$

2. *To find the sine and cosine of the difference of two angles.*

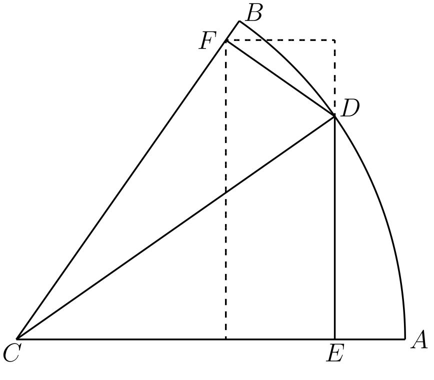
Let

$\theta =$ circular measure of ACB,

$\phi =$ .... of BCD.

Then $\theta - \phi =$ circular measure of ACD.

Draw DE, DF perpendicular to CA, CB.



Then

$$DE = R\sin(\theta-\phi); \qquad CE = R\cos(\theta-\phi);$$

$$CF = R\cos\phi; \qquad FD = R\sin\phi.$$

But DE is the projection of CFD on DE

$$\begin{aligned}&= \text{projection of } CF + \text{projection of } FD\\ &= CF\cos\left(\frac{\pi}{2}-\theta\right) + FD\cos(\pi-\theta)\\ &= CF\sin\theta - FD\cos\theta;\end{aligned}$$

$$\text{i.e. } R\sin(\theta-\phi) = R\cos\phi\sin\theta - R\sin\phi\cos\theta,$$

$$\text{or } \sin(\theta-\phi) = \sin\theta\cos\phi - \cos\theta\sin\phi.$$

Again, CE is the projection of CFD on CA

$$
\begin{aligned}
&= \text{projection of } CF + \text{projection of } FD \\
&= CF\cos\theta + FD\cos\left(\theta - \frac{\pi}{2}\right) \\
&= CF\cos\theta + FD\sin\theta;
\end{aligned}
$$

$$\text{i.e. } R\cos(\theta-\phi) = R\cos\phi\cos\theta + R\sin\phi\sin\theta,$$

$$\text{or } \cos(\theta-\phi) = \cos\phi\cos\theta + \sin\phi\sin\theta.$$

The four expressions for $\sin(\theta \pm \phi)$ and $\cos(\theta \pm \phi)$ are true for all values of θ and ϕ, though the diagrams suppose the angles all acute. They form the fundamental formulæ of Analytical Trigonometry.

APPENDIX. (Note to p. 9.)

To shew that, when the difference between R and r is small, the limit of each radius is very nearly $= \frac{1}{3}(r + 2R)$.

Let $r_1, r_2, \ldots$ be the radii of the circles inscribed in the successive polygons, and $R_1, R_2, \ldots$ of those circumscribed about the same. Let also the limit of both be called r_∞.

Let $R - r = 2\delta$; then $r_1 = \dfrac{R + r}{2} = r + \delta = R - \delta$.

Let

$$r_2 = r_1 + \delta_1; \quad r_3 = r_2 + \delta_2; \quad \text{\&c.}$$

Then

$$r_\infty = r + \delta + \delta_1 + \delta_2 + \text{\&c. } \textit{ad inf.}$$

Also

$$R = r + 2\delta; \quad R_1 = r_1 + 2\delta_1; \quad \text{\&c.}$$

and

$$R = r_1 + \delta; \quad R_1 = r_2 + \delta_1; \quad \text{\&c.}$$

But

$$R_1^2 = r_1 R; \quad \text{or } (r_1 + 2\delta_1)^2 = r_1(r_1 + \delta);$$

$$\therefore 4r_1\delta_1 + 4\delta_1^2 = r_1\delta;$$
$$\therefore \frac{\delta}{4} - \frac{\delta_1^2}{r} < \delta_1 = \frac{\delta}{4} - \frac{\delta_1^2}{r_1} < \frac{\delta}{4};$$
$$\therefore \frac{\delta}{4} - \frac{1}{r}\left(\frac{\delta}{4}\right)^2 < \delta_1 < \frac{\delta}{4}.$$

Similarly

$$\begin{gathered}
\frac{\delta_1}{4} - \frac{1}{r}\left(\frac{\delta}{4^2}\right)^2 < \frac{\delta_1}{4} - \frac{1}{r}\left(\frac{\delta_1}{4}\right)^2 < \delta_2 < \frac{\delta_1}{4}, \\
\frac{\delta_2}{4} - \frac{1}{r}\left(\frac{\delta}{4^3}\right)^2 < \frac{\delta_2}{4} - \frac{1}{r}\left(\frac{\delta_2}{4}\right)^2 < \delta_3 < \frac{\delta_2}{4}, \\
\frac{\delta_3}{4} - \frac{1}{r}\left(\frac{\delta}{4^4}\right)^2 < \frac{\delta_3}{4} - \frac{1}{r}\left(\frac{\delta_3}{4}\right)^2 < \delta_4 < \frac{\delta_3}{4}, \\
\text{\&c.} \qquad\qquad\qquad \text{\&c.}
\end{gathered}$$

$$\therefore \delta + \delta_1 + \delta_2 + \dots \ \textit{ad inf.} < \delta + \frac{\delta}{4} + \frac{\delta_1}{4} + \frac{\delta_2}{4} + \dots \ \textit{ad inf.}$$

and

$$\therefore \frac{3}{4}\{\delta + \delta_1 + \delta_2 + \dots \ \textit{ad inf.}\} < \delta,$$

or

$$\delta + \delta_1 + \delta_2 + \cdots < \frac{4}{3}\,\delta.$$

And

$$\begin{aligned}
\delta + \delta_1 + \delta_2 + \cdots > \delta + \frac{\delta}{4} + \frac{\delta_1}{4} + \ \text{\&c.}\ \textit{ad inf.} \\
- \frac{1}{r}\left\{\left(\frac{\delta}{4}\right)^2 + \left(\frac{\delta}{4^2}\right)^2 + \dots \ \textit{ad inf.}\right\};
\end{aligned}$$

$$\therefore \frac{3}{4}(\delta + \delta_1 + \delta_2 + \dots) > \delta - \frac{\delta^2}{16r} \times \frac{16}{15} > \delta - \frac{\delta^2}{15r};$$

$$\therefore \delta + \delta_1 + \delta_2 + \dots \ \textit{ad inf.} > \frac{4}{3}\,\delta - \frac{4}{45}\,\frac{\delta^2}{r};$$

$$\begin{aligned}\therefore \frac{1}{3}(r + 2R) - \frac{(R - r)^2}{45r} &< r + \frac{4}{3}\,\delta - \frac{4}{45}\,\frac{\delta^2}{r} \\ &< r_\infty \\ &< r + \frac{4}{3}\,\delta < \frac{1}{3}(r + 2R).\end{aligned}$$

So that when, as in the text, $R - r < .00003$ the error in taking the ultimate radius $= \frac{1}{3}(r + 2R)$ is $< \dfrac{.0000000009}{45r}$ which does not affect the tenth decimal place.

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